INTEGRACIÓN DE POTENCIAS DE FUNCIONES TRIGONOMÉTRICAS

Las integrales trigonométricas implican operaciones algebraicas sobre funciones trigonométricas. Algunas identidades trigonométricas que se necesitan en esta sección son las siguientes

Identidades pitagóricas

$\boxed{sen^2 x + \cos^2 x = 1}$ Despejando cada función $\boxed{sen^2 x = 1 - \cos^2 x}$ $\boxed{\cos^2 x = 1 - sen^2 x}$

$\boxed{1 + \tan^2 x = \sec^2 x}$ Despejando cada función $\boxed{1 = \sec^2 x - \tan^2 x}$ $\boxed{\tan^2 x = \sec^2 x - 1}$

$\boxed{1 + \cot^2 x = \csc^2 x}$ Despejando cada función $\boxed{1 = \csc^2 x - \cot^2 x}$ $\boxed{\cot^2 x = \csc^2 x - 1}$

Identidades del ángulo medio

$\boxed{sen^2 x = \frac{1 - \cos 2x}{2}}$ $\boxed{\cos^2 x = \frac{1 + \cos 2x}{2}}$

Paso de producto a suma

$$senmx.sennx = \frac{1}{2}\left(\cos[(m-n)x] - \cos[(m+n)x]\right)$$
$$senmx.\cos nx = \frac{1}{2}\left(sen[(m-n)x] + sen[(m+n)x]\right)$$
$$\cos mx.\cos nx = \frac{1}{2}\left(\cos[(m-n)x] + \cos[(m+n)x]\right)$$

Para ángulos opuestos

$\boxed{sen(-x) = -sen(x)}$ $\boxed{\cos(-x) = \cos(x)}$ $\boxed{\tan(-x) = -\tan(x)}$

$\boxed{\csc(-x) = -\csc t(x)}$ $\boxed{\sec(-x) = \sec(x)}$ $\boxed{\cot(-x) = -\cot(x)}$

Suma y diferencia de ángulos

$$sen(x \pm y) = sen(x)\cos(y) \pm \cos(x)sen(y)$$
$$\cos(x \pm y) = \cos(x)\cos(y) \mp sen(x)sen(y)$$
$$\tan(x \pm y) = \frac{\tan(x) \pm \tan(y)}{1 \mp \tan(x)\tan(y)}$$

Una vez realizadas las transformaciones trigonométricas, el integrando queda listo para aplicar integración por sustitución. En algunos casos se debe recurrir a la integración por partes.

Resolver los siguientes ejercicios

Ejercicio	Respuesta	Ejercicio	Respuesta
1. $\int \cos^3 x dx$	$senx - \frac{sen^3 x}{3} + c$	2. $\int sen^5 x dx$	$-\cos x + \frac{2\cos^3 x}{3} - \frac{\cos^5 x}{5} + c$
3. $\int sen^2 x dx$	$\frac{x}{2} - \frac{sen2x}{4} + c$	4. $\int \cos^2 x dx$	$\frac{x}{2} + \frac{sen2x}{4} + c$
5. $\int \cos^4 x dx$	$\frac{3x}{8} + \frac{sen2x}{4} + \frac{sen4x}{32} + c$	6. $\int sen^4 \theta d\theta$	$\frac{3\theta}{8} - \frac{sen2\theta}{4} + \frac{sen4\theta}{32} + c$
7. $\int sen^2 2x dx$	$\frac{x}{2} - \frac{sen4x}{8} + c$	8. $\int sen^4 2\theta d\theta$	$\frac{3\theta}{8} - \frac{sen4\theta}{8} + \frac{sen8\theta}{64} + c$
9. $\int \cos^2 \frac{x}{2} dx$	$\frac{x}{2} + \frac{senx}{2} + c$	10. $\int sen^3 x \cos^4 x dx$	$-\frac{\cos^5 x}{5} + \frac{\cos^7 x}{7} + c$
11. $\int sen^4 x \cos x dx$	$\frac{sen^5 x}{5} + c$	12. $\int sen^2 x \cos^3 x dx$	$\frac{sen^3 x}{3} - \frac{sen^5 x}{5} + c$
13. $\int sen^5 x \cos^2 x dx$	$\frac{\cos^3 x}{3} - \frac{2\cos^5 x}{5} + \frac{\cos^7 x}{7} + c$	14. $\int \cos^3 4x sen4x dx$	$-\frac{\cos^4 4x}{16} + c$
15. $\int sen^4 5x \cos 5x dx$	$\frac{sen^5 5x}{25} + c$	16. $\int sen^2 x \cos^2 x dx$	$\frac{x}{8} - \frac{sen4x}{32} + c$
17. $\int sen^2 3x \cos^2 3x dx$	$\frac{x}{8} - \frac{sen12x}{96} + c$	18. $\int sen^3 x \sqrt{\cos x} dx$	$-\frac{2\sqrt{\cos^3 x}}{3} + \frac{2\sqrt{\cos^7 x}}{7} + c$
19. $\int \frac{\cos^3 3x}{\sqrt[3]{sen3x}} dx$	$\frac{\sqrt[3]{sen^2 3x}}{2} - \frac{\sqrt[3]{sen^8 x}}{8} + c$	20 $\int sen^3 2t \sqrt{\cos 2t} dt$	$-\frac{\sqrt{\cos^3 2t}}{3} + \frac{\sqrt{\cos^7 2t}}{7} + c$
21 $\int sen^{1/2} 2\alpha \cos^3 2\alpha d\alpha$	$\frac{\sqrt{sen^3 2\alpha}}{3} - \frac{\sqrt{sen^7 2\alpha}}{7} + c$	22. $\int sen^3 x \cos^{-4} x dx$	$\frac{\sec^3 x}{3} - \sec x + c$
23. $\int \cos^3 3\theta sen^{-2} 3\theta d\theta$	$-\frac{\csc 3\theta}{3} - \frac{sen3\theta}{3} + c$	24. $\int sen5x \cos 4x dx$	$-\frac{\cos x}{2} - \frac{\cos 9x}{18} + c$
25. $\int sen4y \cos 5y dy$	$\frac{\cos y}{2} - \frac{\cos 9y}{18} + c$	26. $\int \cos y \cos 4y dy$	$\frac{sen3y}{6} + \frac{sen5y}{10} + c$
27. $\int sen3t sent dt$	$\frac{sen2t}{4} - \frac{sen4t}{8} + c$	28. $\int sen3y \cos 5y dy$	$\frac{\cos 2y}{4} - \frac{\cos 8y}{16} + c$
29. $\int \cos 3t \cos t dt$	$\frac{sen2t}{4} + \frac{sen4t}{8} + c$	30. $\int \tan^2 x dx$	$\tan x - x + c$
31. $\int \cot^2 x dx$	$-\cot x - x + c$	32. $\int \tan^3 x dx$	$\frac{\tan^2 x}{2} + \ln\lvert\cos x\rvert + c$
33. $\int \cot^4 x dx$	$-\frac{\cot^3 x}{3} + \cot x + x + c$	34. $\int \cot^4 3x dx$	$-\frac{\cot^3 3x}{9} - \frac{\cot 3x}{3} + x + c$
35. $\int \tan^4 x dx$	$\frac{\tan^3 x}{3} - \tan x + x + c$	36. $\int \cot^3 2t dt$	$-\frac{\cot^2 2t}{4} - \frac{\ln\lvert sen2t\rvert}{4} + c$

37. $\int \tan^6 3x dx$	$\frac{\tan^5 3x}{15}-\frac{\tan^3 3x}{9}+\frac{\tan 3x}{3}-x$	38. $\int \cot^5 2x dx$	$-\frac{\cot^4 2x}{8}+\frac{\cot^2 2x}{4}+\frac{\ln sen2x}{2}$
39. $\int \sec^4 x dx$	$\frac{\tan^3 x}{3}+\tan x + c$	40. $\int \csc^6 x dx$	$-\frac{\cot^5 x}{5}-\frac{2\cot^3 x}{3}+\cot x + c$
41. $\int \sec^3 x dx$	$\frac{\sec x \tan x}{2}+\frac{\ln\lvert\sec x+\tan x\rvert}{2}$	42. $\int \sec^6 x dx$	$\frac{\tan^5 x}{5}+\frac{2\tan^3 x}{3}+\tan x + c$
43. $\int \csc^3 x dx$	$-\frac{\csc x \cot x}{2}+\frac{\ln\lvert\csc x-\cot x\rvert}{2}$	44. $\int \tan^5 3\theta d\theta$	$\frac{\tan^4 3\theta}{12}-\frac{\tan^2 3\theta}{6}-\frac{\ln\lvert\cos 3\theta\rvert}{3}$
45. $\int \tan^6 x \sec^4 x dx$	$\frac{\tan^9 x}{9}+\frac{\tan^7 x}{7}+c$	46. $\int \tan^5 x \sec^7 x dx$	$\frac{\sec^{11} x}{11}-\frac{2\sec^9 x}{9}+\frac{\sec^7 x}{7}+c$
47. $\int \tan^5 x \sec^4 x dx$	$\frac{\tan^8 x}{8}+\frac{\tan^6 x}{6}+c$	48 $\int \tan^5 x \sec^5 x dx$	$\frac{\sec^9 x}{9}-\frac{2\sec^7 x}{7}+\frac{\sec^5 x}{5}+c$
49. $\int \tan^3 x \sec^9 x dx$	$\frac{\sec^{11} x}{11}-\frac{\sec^9 x}{9}+c$	50 $\int \tan^3 2x \sec^5 2x dx$	$\frac{\sec^7 2x}{14}-\frac{\sec^5 2x}{10}+c$
51. $\int \sec^4 3x \tan^3 3x dx$	$\frac{\tan^6 3x}{18}+\frac{\tan^4 3x}{12}+c$	52. $\int \tan^{-3} x \sec^4 x dx$	$\ln\lvert\tan x\rvert-\frac{\cot^2 x}{2}+c$
53. $\int \tan^3 x \sec^{-1/2} x dx$	$\frac{2\sqrt{\sec^3 x}}{3}+\frac{2}{\sqrt{\sec x}}+c$	54. $\int \frac{\tan^3 \theta}{\cos^4 \theta} d\theta$	$\frac{\tan^6 \theta}{6}+\frac{\tan^4 \theta}{4}+c$
55. $\int \frac{\sec x}{\tan^2 x} dx$	$-\csc x + c$	56. $\int \cot^2 3x \csc^4 3x dx$	$-\frac{\cot^5 3x}{15}-\frac{\cot^3 3x}{9}+c$
57. $\int \cot^3 \alpha \csc^3 \alpha d\alpha$	$-\frac{\csc^5 \alpha}{5}+\frac{\csc^3 \alpha}{3}+c$	58. $\int \csc^4 \alpha \cot^6 \alpha d\alpha$	$-\frac{\cot^9 \alpha}{9}-\frac{\cot^7 \alpha}{7}+c$
59. $\int \csc^2 3x \cot 3x dx$	$-\frac{\cot^3 3x}{9}+c$	60. $\int \frac{\cot^3 x}{\csc x} dx$	$-\csc x - senx + c$

Integrales de potencias de funciones trigonométricas	
$\int sen^n xdx = \int (1-\cos^2 x)^{\frac{n-1}{2}} senxdx$ $\int \cos^n xdx = \int (1-ses^2 x)^{\frac{n-1}{2}} \cos xdx$ n = impar	$= \int \left(\frac{1-\cos 2x}{2}\right)^{\frac{n}{2}} dx$ $= \int \left(\frac{1+\cos 2x}{2}\right)^{\frac{n}{2}} dx$ n = par
$\int sen^n x \cos^m xdx = \int (1-\cos^2 x)^{\frac{n-1}{2}} \cos^m xsenxdx$ $\int sen^n x \cos^m xdx = \int (1-sen^2 x)^{\frac{m-1}{2}} sen^n x \cos xdx$	n = impar m = impar
$\int \tan^n xdx = \int \tan^{n-2} x(\sec^2 x - 1)dx$ $\int \cot^n xdx = \int \cot^{n-2} x(\csc^2 x - 1)dx$	n = entero positivo n = entero positivo
$\int \sec^n xdx = \int (\tan^2 x + 1)^{\frac{n-2}{2}} (\sec^2 x)dx$ $\int \csc^n xdx = \int (\cot^2 x + 1)^{\frac{n-2}{2}} (\csc^2 x)dx$ n = entero positivo par	$u = \sec^{n-2} x \quad dv = \sec^2 xdx$ $u = \csc^{n-2} x \quad dv = \csc^2 xdx$ n = entero positivo impar
$\int \tan^n x \sec^m xdx = \int \tan^n x(\tan^2 x + 1)^{\frac{m-2}{2}} (\sec^2 xdx)$ $= \int (\sec^2 x - 1)^{\frac{n-1}{2}} \sec^{m-1} x(\sec x \tan xdx)$ $= \int (\sec^2 x - 1)^{\frac{n}{2}} \sec^m xdx$	*m = par* *n = par o impar* *m = impar* *n = impar* *m = impar* *n = par*
$\int \cot^n x \csc^m xdx = \int \cot^n x(\cot^2 x + 1)^{\frac{m-2}{2}} (\csc^2 xdx)$ $= \int (\csc^2 x - 1)^{\frac{n-1}{2}} \csc^{m-1} x(\csc x \cot xdx)$ $= \int (\csc^2 x - 1)^{\frac{n}{2}} \csc^m xdx$	*m = par* *n = par o impar* *m = impar* *n = impar* *m = impar* *n = par*